

Write your name here

Surname	Other names
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Pearson

Edexcel GCE

Centre Number

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Candidate Number

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A level Mathematics

Practice Paper

Pure Mathematics - Numerical methods

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You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 102.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$.

(a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x} \quad (3)$$

(b) On a pair of axes, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y -axis and state the equation of any asymptotes.

(4)

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = \left(-2 - e^{4x_n}\right)^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

(d) Calculate the values of x_1 and x_2 , giving your answers to 5 decimal places.

(2)

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C .

(2)

(Total 12 marks)

2. $f(x) = x^3 + 3x^2 + 4x - 12$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{3+x}\right)}, \quad x \neq -3. \quad (3)$$

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{3+x_n}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1 , x_2 and x_3 . (3)

The root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places. (3)

(Total 9 marks)

3. $g(x) = e^{x-1} + x - 6$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \quad x < 6. \quad (2)$$

The root of $g(x) = 0$ is α .

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2.$$

is used to find an approximate value for α .

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places. (3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places. (3)

(Total 8 marks)

4. $f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}.$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$. (5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5}e^{-x}$. (1)

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. (3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer. (2)

(Total 11 marks)

5.

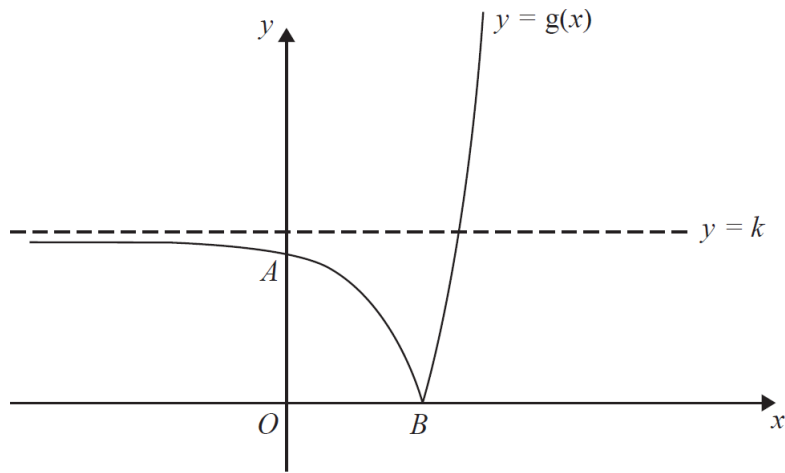


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = g(x)$, where

$$g(x) = |4e^{2x} - 25|, \quad x \in \mathbb{R}.$$

The curve cuts the y -axis at the point A and meets the x -axis at the point B . The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

(a) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A ,

(ii) the exact x coordinate of the point B ,

(iii) the value of the constant k .

(5)

The equation $g(x) = 2x + 43$ has a positive root at $x = \alpha$.

(b) Show that α is a solution of $x = \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$.

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2} \ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α .

(c) Taking $x_0 = 1.4$, find the values of x_1 and x_2 . Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

(Total 11 marks)

6.

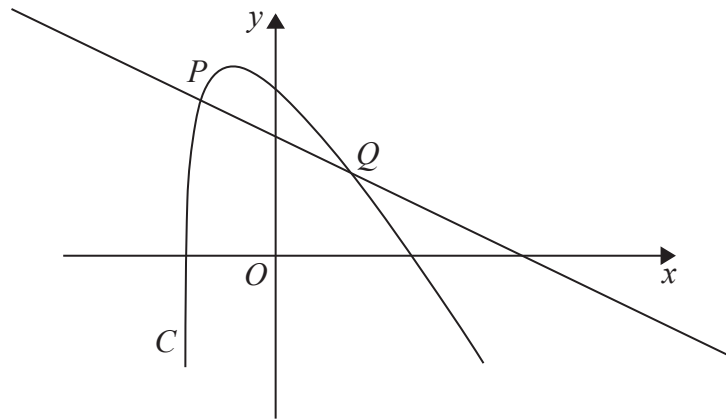


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \quad x > -2.5$$

The point P with x coordinate -2 lies on C .

- (a) Find an equation of the normal to C at P . Write your answer in the form $ax + by = c$, where a , b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q , as shown in Figure 2.

- (b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q .

- (c) Taking $x_1 = 2$, find the values of x_2 and x_3 , giving each answer to 4 decimal places.

(2)

(Total 10 marks)

7.

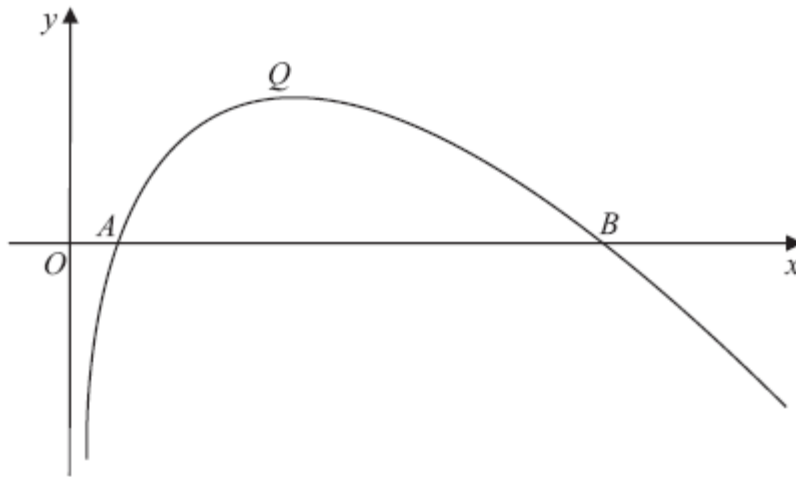


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \quad x > 0.$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 3.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}. \quad (3)$$

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)

(Total 13 marks)

8.

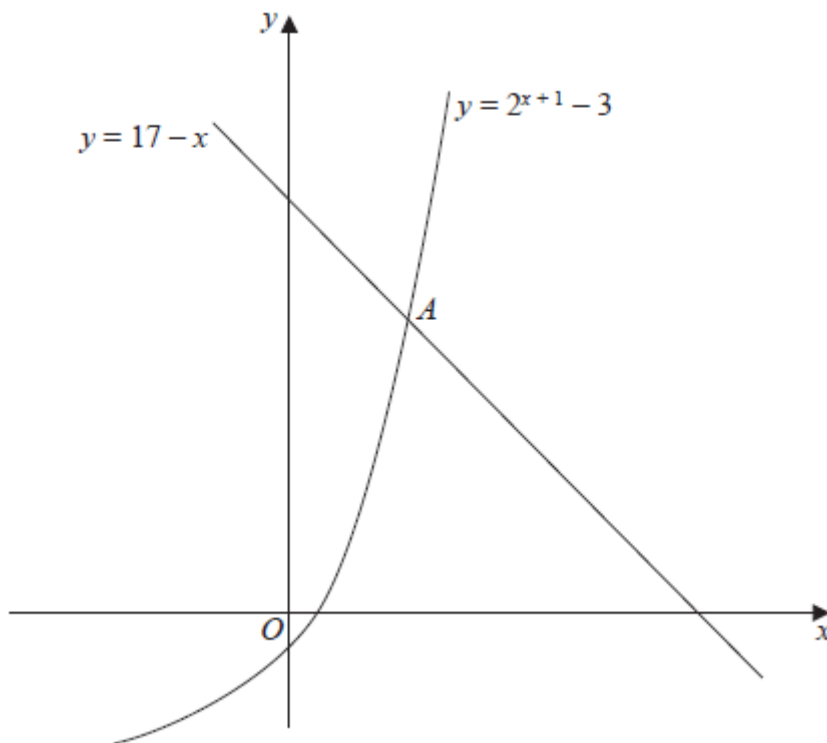


Figure 4

Figure 4 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A .

(a) Show that the x -coordinate of A satisfies the equation

$$x = \frac{\ln(20-x)}{\ln 2} - 1. \quad (3)$$

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20-x_n)}{\ln 2} - 1, \quad x_0 = 3,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

(c) Use your answer to part (b) to deduce the coordinates of the point A , giving your answers to one decimal place. (2)

(Total 8 marks)

9.

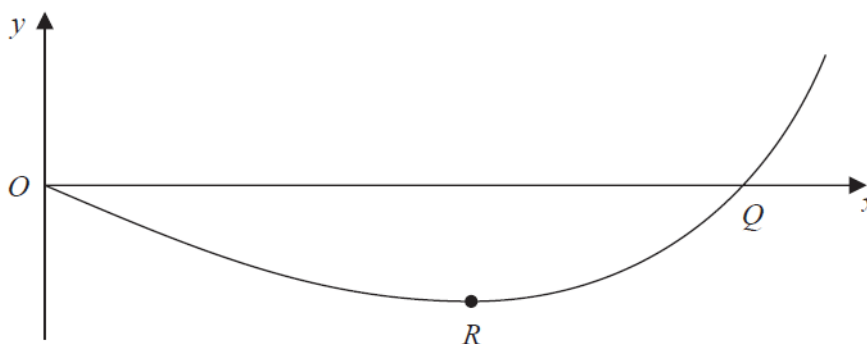


Figure 5

Figure 5 shows a sketch of part of the curve with equation

$$y = 2 \cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x -axis at the point Q and has a minimum turning point at R .

(a) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x \sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n \sin\left(\frac{1}{2}x_n^2\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

(Total 8 marks)

10. $f(x) = x^2 - 3x + 2 \cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi.$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$. (2)

The curve with equation $y = f(x)$ has a minimum point P .

(b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}. \quad (4)$$

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2,$$

find the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. (3)

(d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places. (3)

(Total 12 marks)

TOTAL FOR PAPER: 102 MARKS